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Market**

**Vasileios Zikos**

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Dept Economics  
Loughborough University  
Loughborough  
LE11 3TU United Kingdom  
Tel: + 44 (0) 1509 222701  
Fax: + 44 (0) 1509 223910  
<http://www.lboro.ac.uk/departments/ec>



# **Equilibrium and Optimal R&D Roles in a Mixed Market<sup>\*</sup>**

Vasileios Zikos

Department of Economics

Loughborough University

Loughborough LE11 3TU

United Kingdom

V.Zikos@lboro.ac.uk

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## **Abstract**

This is the first paper to investigate the timing of the R&D decisions in a mixed market. Considering a model in which a public firm competes against a private one, we examine the desirable (welfare-maximizing) and the equilibrium R&D role of the public firm. Our results suggest that from a social point of view, the public firm should carry out its investment as a Stackelberg follower. Using the observable delay game of Hamilton and Slutsky [Games and Economic Behavior 2 (1990) 29], we show that the public firm may play this desirable role.

*Keywords:* Endogenous timing, R&D, Stackelberg, mixed market.

*JEL Classification:* L13, L31, L32.

# 1 Introduction

Strategic R&D competition between public and private firms has become an increasingly active field of interest. For instance, in the Norwegian oil industry the state-owned company Statoil competes against the private firms Norske Shell (or Shell Technology) and Exxon (see e.g. Pal, 1998). These companies – with Exxon to a lesser extent – initiate large technological programmes.<sup>1</sup> In that and related contexts, an alternative sequence in the firms' R&D decisions may affect the market outcome in important ways. Surprisingly though, issues of timing in the R&D moves of firms operating in mixed markets have not been considered so far by the literature.

Previous studies have investigated sequential-move models with an output market focus and showed that when the firms' decision is endogenised the outcome is in general ambiguous. Pal (1998) showed that depending on the number of private firms (and the time periods the game lasts), the public firm can either be the leader or the follower in equilibrium.<sup>2</sup> More recently, Matsumura (2003) extended Pal's analysis by allowing for foreign firms. In the context of a duopoly market, where the output decision process can last up to two time periods, he showed that the equilibrium role of the public firm (which coincides with its desirable role) is that of an output-setting leader. However, Pal (1998) and Matsumura (2003) did not allow for R&D investments. Nishimori and Ogawa (2002) focused on this issue, though under a different research direction, and found that deregulation of a former public monopoly induces a reduction in the public firm's investments.<sup>3</sup> Finally, Gil Moltó, Poyago-Theotoky and Zikos (2006) investigated

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<sup>1</sup>Statoil was very active in R&D aimed at the development of fuel cells and related hydrogen technologies. Now, it is mainly investing on the development of a dual cycle energy production system based on fuel cells. Similarly, Norske Shell is a major investor in energy systems based on SOFC (Solid Oxide Fuel Cells); see Godø et al. (2003) for a more detailed discussion.

<sup>2</sup>Jacques (2004) and Lu (2007) qualified Proposition 4.1 in Pal (1998) by identifying under which circumstances it remains valid and by showing the existence of two additional equilibria.

<sup>3</sup>For earlier contributions see Delbono and Denicoló (1993), Nett (1994) and Poyago-Theotoky (1998).

the role of technology policy towards R&D both before and after privatization. They showed that privatization leads to a lower R&D subsidy and reductions in the level of social welfare.

These papers have shed considerable light on aspects of strategic interaction in mixed markets. However, a number of issues still await investigation and these indeed form the subject of the present paper. In the context of a mixed duopoly (where a public firm competes against a private one), the first objective of this paper is to examine the desirable R&D role of the public firm. By considering a two-stage Cournot model – building upon d’Aspremont and Jacquemin (1988) – as a benchmark case, we are in position to disentangle the effect on consumer surplus and the firms’ profits implied by a change in the sequence of R&D moves. This, in turn, will determine the regime, which is most desirable from a social perspective. The same influence on social welfare can be uncovered by analyzing the market failures in the cases when the public firm retains the leader or follower position. The key point here is the co-existence of distortions in the R&D and in the output markets, which are indeed carefully analyzed.

A classification of welfare levels is essential in order to determine the desirable R&D role of the public firm. As a next step, it would be reasonable to explore which roles will actually be chosen in equilibrium. For this purpose, we adopt the observable delay game of Hamilton and Slutsky (1990).<sup>4</sup> In this game, the firms first choose when to invest in R&D (and commit to this decision); firms then make their R&D investments (at the same stage or at different stages) and finally, they compete in quantities. The study of this game will reveal whether the desirable allocation of R&D roles can, in fact, arise in

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<sup>4</sup>In their seminal contribution on endogenous timing, Hamilton and Slutsky (1990) also presented an alternative game; so-called action commitment game. According to this game it is possible that in a mixed strategy equilibrium firms choose output levels other than those implied by the Cournot or Stackelberg games. Hence, the observable delay game is more suitable to discuss the endogeneity of the firms’ roles (see also Matsumura, 2003).

equilibrium. In turn, this will indicate whether ‘coordination’ of the R&D activity among firms can be achieved without any form of intervention.

The rest of the paper is organised as follows. Section 2 presents the model and offers an explanation of the observable delay game. Section 3 then derives the Nash equilibrium for the Cournot and the Stackelberg games of fixed timing. The next section turns to a comparison of the results obtained and presents the desirable R&D role of the public firm. Section 5 investigates its equilibrium role and finally, section 6, draws the conclusions.

## 2 The model

Consider an industry in which a state-owned public firm competes with a private firm. The inverse demand function is assumed linear of the standard form  $P(Q) = A - Q$ , where  $Q = q_0 + q_1$  and  $A \geq Q$ ; let  $P$  denote the price of the homogeneous product and  $q_0$ ,  $q_1$ , the quantities of the public and the private firm, respectively. Suppose further that all firms engage in cost-reducing (process) R&D under a fully efficient patent system<sup>5</sup> (i.e., spillovers are equal to zero). All firms have identical cost functions of the form  $C_i(x_i, q_i) = (c - x_i)q_i + kq_i^2$ , where  $A > c$ ,  $k > 0$  and  $x_i$  denotes the R&D investment of firm  $i$ , for  $0 < x_i \leq c$ ,  $i \in \{0, 1\}$ .<sup>6</sup> Moreover, the R&D cost borne by firm  $i$  is quadratic, so as to capture diminishing returns in the rate of R&D investment  $x_i$ ,  $\Gamma_i(x_i) = x_i^2$ .

The profit function of firm  $i$  is given by:

$$\pi_i = P(Q)q_i - C_i(x_i, q_i) - \Gamma_i(x_i), \quad i \in \{0, 1\}. \quad (1)$$

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<sup>5</sup>Admittedly, this assumption is rather special and is made for the purpose of tractability. In future work, we aim at exploring the implications of relaxing it.

<sup>6</sup>As in the seminal framework of d’Aspremont and Jacquemin (1988), the marginal cost curve shifts downwards by the amount of R&D output, while its slope is not affected, i.e.,  $mc_i = (c - x_i) + 2kq_i$ . Moreover, the presence of a quadratic term in the (total) cost function is part of a routinely formulation in mixed oligopolies that serves the purpose of ruling out the uninteresting case of a public monopoly (through the introduction of diminishing returns in production).

Social welfare, defined as the sum of consumer and producer surplus, is given by:

$$SW = \frac{1}{2}Q^2 + \sum_{i=0}^1 [P(Q)q_i - C_i(x_i, q_i) - \Gamma_i(x_i)]. \quad (2)$$

We now turn to present the *observable delay game* of Hamilton and Slutsky (1990).

In the first stage, firms simultaneously and independently announce at which period they will choose their R&D levels (and commit to this decision).<sup>7</sup> Each firm then chooses its R&D level and finally, firms compete in quantities.

A more detailed discussion of the game – based on an excellent exposition by Matsumura (2003) – is in order. In stage one ( $t = 1$ ), firms choose simultaneously and independently  $t_i \in (2, 3)$ . This means that if  $t_i = 2$ , firm  $i$  will choose its R&D level in period 2. (Similarly for  $t_i = 3$ .) Following their decisions firms observe  $t_1$  and  $t_2$ . In stage two ( $t = 2$ ), the firm that has chosen  $t_i = 2$  makes its R&D investment, while knowing when the other firm will choose its own R&D level. Hence, if  $t_i = 2$  for all  $i$ , a simultaneous-moves (Cournot) game in R&D arises; however, if  $t_i \neq t_j$ ,  $i \neq j$ ,  $i, j \in \{0, 1\}$ , then the game is sequential.<sup>8</sup> Finally, a Cournot game in output is played. Our objective is to solve for the Subgame Perfect Nash Equilibrium (SPNE) of this extended game by backward induction.<sup>9</sup>

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<sup>7</sup>As noted by Pal (1998), the assumption of “commitment” is not restrictive in the sense that firms have no incentive to deviate from their decision in a later time period.

<sup>8</sup>In case that  $t_i = 2$  and  $t_j = 3$ , this means that firm  $j$  *delays* its R&D choice (until period 3), while firm  $i$  chooses its R&D level in the earlier time period 2. The terminology “observable delay game” is due to the fact that at the end of the first stage each firm observes when its rival will move and indeed whether it will delay investing in R&D.

<sup>9</sup>To check the robustness of our results one possible extension would be to consider the case when firms compete in prices (Bertrand competition) rather than in quantities (Cournot competition). We find that in this case, a complete characterization of the SPNE outcomes becomes quite complex and several questions cannot be thoroughly answered. However, it is clear that a comparison between the two models will depend critically on the difference in the nature of strategic interaction, which will presumably alter (some of) the conclusions of our analysis.

### 3 Games of fixed timing

#### 3.1 The Cournot R&D game

According to the timing of this game, in the last stage each firm chooses its quantity  $q_i$  to maximize its objective. From the first order conditions (focs) of this problem, we obtain the Cournot-Nash equilibrium quantities of the public and the private firm:

$$q_0^c(x_0, x_1) = \frac{(2k+1)(A-c) + 2(1+k)x_0 - x_1}{1+6k+4k^2}, \quad (3)$$

$$q_1^c(x_0, x_1) = \frac{2k(A-c) + (2k+1)x_1 - x_0}{1+6k+4k^2}, \quad (4)$$

where the superscript ‘ $c$ ’ refers to the case of Cournot R&D competition.

In the first stage, firms choose their R&D levels  $x_i$  anticipating how this may affect competition at the output selection stage. Substituting (3) and (4) into (1) and (2) and taking the first order condition (foc) with respect to  $x_i$ , this gives rise to the following R&D reaction functions:<sup>10</sup>

$$r_0^c(x_1) = \frac{(1+6k+16k^2+8k^3)(A-c) - 2(1+4k+2k^2)x_1}{-1+10k+68k^2+88k^3+32k^4}, \quad (5)$$

$$r_1^c(x_0) = \frac{(1+3k+2k^2)[2k(A-c) - x_0]}{k(7+36k+44k^2+16k^3)}. \quad (6)$$

Notice that the R&D reaction curves are negatively sloped (i.e., R&D is a strategic substitute). This amounts to saying that if firm  $i$  increases its own R&D level, this will lead to a reduction in the R&D level of firm  $j$ , for  $i \neq j$ ,  $i, j \in \{0, 1\}$ . Solving the system

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<sup>10</sup>The second order condition (soc) for the public firm is  $-\frac{-1+10k+68k^2+88k^3+32k^4}{(1+6k+4k^2)^2} < 0$  and for the private firm  $-\frac{2k(7+36k+44k^2+16k^3)}{(1+6k+4k^2)^2} < 0$ . Moreover, the stability condition for the public firm reads as  $\left| \frac{2(1+4k+2k^2)}{-1+10k+68k^2+88k^3+32k^4} \right| < 1$  and for the private firm  $\left| \frac{(1+3k+2k^2)}{k(7+36k+44k^2+16k^3)} \right| < 1$ . Indeed, all conditions are fulfilled.



of (5) and (6), we obtain the equilibrium R&D levels:

$$x_0^c = \frac{k(3 + 14k + 8k^2)(A - c)}{-2 + 3k + 54k^2 + 80k^3 + 32k^4}, \quad (7)$$

$$x_1^c = \frac{(-1 + k + 10k^2 + 8k^3)(A - c)}{-2 + 3k + 54k^2 + 80k^3 + 32k^4}. \quad (8)$$

Then from (3), (4), (7) and (8), the SPNE solutions of the entire game follow (see Appendix A).

### 3.2 The Stackelberg R&D game with a public leader

We now assume that the public firm sets its R&D level as a Stackelberg leader. In the last stage of the game, the expressions for equilibrium output are given by (3) and (4). In the preceding stage, the private follower maximizes profits with respect to  $x_1$ . Then the public leader chooses its own R&D level to maximize welfare, anticipating the reaction function of the follower as in (6). This yields the following investment:<sup>11</sup>

$$x_0^l = \frac{k\Theta(A - c)}{\Omega}, \quad (9)$$

where the superscript ‘ $l$ ’ stands for the case of the public R&D leader;  $\Theta = 17 + 194k + 684k^2 + 952k^3 + 576k^4 + 128k^5$  and  $\Omega = -16 - 57k + 394k^2 + 2604k^3 + 5528k^4 + 5536k^5 + 2688k^6 + 512k^7$ . Substitutions reveal the equilibrium values of the game (see Appendix B).

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<sup>11</sup>The soc reads as  $-\frac{H}{k(7+36k+44k^2+16k^3)^2} < 0$ , where  $H = -16 - 57k + 394k^2 + 2604k^3 + 5528k^4 + 5536k^5 + 2688k^6 + 512k^7$ .

### 3.3 The Stackelberg R&D game with a private leader

We now turn to the case where the private firm retains the R&D leader position. Following the same procedure as for the previous games, we obtain the second stage reaction function for the public follower as in (5). The private firm then maximizes profits taking into account (5). The solution to this yields:<sup>12</sup>

$$x_1^f = \frac{(1 + 6k + 4k^2)^2(-1 - k + 16k^2 + 16k^3)(A - c)}{k\Gamma}, \quad (10)$$

where ‘ $f$ ’ denotes the case of the public R&D follower and  $\Gamma = -41 - 212k + 436k^2 + 4624k^3 + 10608k^4 + 10944k^5 + 5376k^6 + 1024k^7$ . Substitutions reveal the equilibrium values of the game, which are given in the Appendix C.

## 4 Comparison

In this section, we compare the results of the Stackelberg and Cournot models. The following Proposition contains an ordering of the R&D and output levels:<sup>13</sup>

**Proposition 1** *For all  $k > 0$ , the ordering of R&D investment and output for the public and the private firm under the different structures  $\rho = c, f, l$  in the timing of R&D decisions are as follows:*

- (i)  $x_0^c > x_0^f > x_0^l$ , (ii)  $x_1^f > x_1^l > x_1^c$ ,
- (iii)  $q_0^c > q_0^f > q_0^l$ , (iv)  $q_1^f > q_1^l > q_1^c$ .

According to part (iii), the public firm’s presence as a Stackelberg leader is accompanied by the lowest output among all cases. This result, not new to the literature, is

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<sup>12</sup>The soc is  $-\frac{2k\mu}{(-1+10k+68k^2+88k^3+32k^4)^2} < 0$ , where  $\mu = -41 - 212k + 436k^2 + 4624k^3 + 10608k^4 + 10944k^5 + 5376k^6 + 1024k^7$ .

<sup>13</sup>All proofs are relegated to the Appendix D.

in line with the predictions of the seminal paper by De Fraja and Delbono (1989) and may provide us with an understanding of the ranking applying to R&D. That is, when the public firm is the leader, it recognizes that the R&D reaction function of the private follower is downward sloping. On the grounds of this observation and taking into account that the private firm is more efficient (i.e., it has a lower marginal and total cost), it is optimal for the public firm to reduce its investment relative to the case of Cournot competition. (By doing so, the public firm may partially correct an important failure of a Cournot mixed market – the inefficiency in the distribution of equilibrium cost – arising from the difference in the firms’ objectives.) Moreover, we obtain the intuitive outcome that the public firm invests more in R&D as a Cournot player rather than when it engages in a Stackelberg game at the follower position. The reverse ranking applies to the private competitor for the case of Cournot competition, i.e., private R&D is the lowest. As might also be expected, the largest investment is attained when the private firm is a Stackelberg leader.

As for total R&D and output, the following result is largely a consequence of Proposition 1.

**Corollary 1** *Stackelberg leadership in R&D by the public firm generates the smallest total investment, whereas the largest investment is associated with the public firm being a Stackelberg R&D follower; i.e., the ordering  $X^l < X^c < X^f$  holds. The same ordering applies to the production of output.*

A classification of consumer surplus is then immediate, since process R&D reduces marginal cost and expands output, thus leading to a larger consumer surplus.<sup>14</sup>

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<sup>14</sup>Notice that process R&D implies an indirect effect on (gross) consumer surplus via a reduction in marginal cost, which is followed by an decrease in price. By way of contrast, product R&D raises quality and indeed exerts a direct effect on consumers’ surplus, as quality enters directly the consumers’ utility function (see e.g. Symeonidis, 2003).

**Corollary 2** *Consumer surplus is ordered as follows:*

$$CS^f > CS^c > CS^l.$$

We now turn to compare the private firm's profit across the three regimes. Thus, we have the following:

**Proposition 2** *The private firm's profits are the largest when it moves as a Stackelberg R&D follower; i.e., the ordering  $\pi_1^l > \pi_1^f > \pi_1^c$  holds.*

Proposition 2 shows that Stackelberg R&D leadership by the public firm increases the private firm's profits (compared with the other regimes). The reason is that in this case, the public firm's (welfare-maximizing) behaviour constrains total output, which in turn expands the private firm's profits.

Next, we are in position to identify the relationship between the order of the firms' R&D moves and producer surplus. This will assist us to classify welfare among the three different situations.

**Proposition 3** *Producer surplus attains its highest value when the public firm acts as a Stackelberg leader in choosing its R&D investment, while the lowest value corresponds to the Cournot conjecture. That is,*

$$PS^l > PS^f > PS^c.$$

Proposition 3 clarifies that when the public firm retains leader position in R&D, the firms' market power and so their profits are the highest. Consequently, consumer surplus is the lowest (Corollary 2).

On the grounds of Corollary 2 and Proposition 3, we proceed to present our main results starting with the welfare comparisons among the three regimes. Pal (1998) showed

that in an output setting duopoly the public firm may increase welfare by acting as a Stackelberg leader.<sup>15</sup> Matsumura (2003) reached a similar conclusion for a mixed market, where the private firm is foreign. However, in some other instances these results may not hold (see e.g. Nishimori and Ogawa, 2005, p. 285). In a similar spirit, the next Proposition shows that in an R&D setting duopoly the preferable role for the public firm is that of the follower in choosing its R&D investment.

**Proposition 4** *The ordering of welfare levels under the different structures  $\rho = c, f, l$  in the timing of R&D setting are as follows:*

$$SW^f > SW^l > SW^c.$$

Proposition 4 indicates the desirable role of the public firm in the R&D game. More precisely, the highest welfare is attained when the public firm acts as a Stackelberg follower (while the private firm is the leader). The following **remarks** may be useful in understanding the result: (i) when the public firm is the *leader* consumer surplus is the lowest, while producer surplus is the highest; however, (ii) when the public firm invests in R&D as a *follower* consumer surplus is the highest, but producer surplus is intermediate. In what follows (i) and (ii) are compared to the case of Cournot competition. In case (i), it turns out that the increase in producer surplus ( $PS^l > PS^f > PS^c$ ) relative to the Cournot case is large enough and so outweighs the decrease in consumer surplus ( $CS^l < CS^c$ ); this implies an expansion in the level of total welfare ( $SW^c < SW^l$ ). On the other hand, in case (ii), we have an increase in both consumer ( $CS^f > CS^c$ ) and producer surplus ( $PS^f > PS^c$ ); indeed, so large an increase that total welfare improves

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<sup>15</sup>Pal (1998) showed that the situation in which the public firm is a Stackelberg leader in output, whereas the private firm a follower, can be sustained as a SPNE if the number of private firms in the industry is  $n \leq 2$  and the game lasts for two time periods. (In the first period both firms announce when they will produce and in the second period they choose their output levels.) If the game lasts for more than two time periods there are two SPNE for the duopoly case. In one of those equilibria reported by Jacques (2004), the public firm produces in period 1 and the private firm produces in the last period.

more than in the case where the public firm retains the R&D role of a Stackelberg leader ( $SW^c < SW^l < SW^f$ ).

A second line of reasoning referring to the market failures at work is in order. As is well-known, a mixed market suffers from two main distortions: *underproduction* due to imperfect competition and *inefficiency in the allocation of production costs* resulting from the firms' divergent objectives. It is worth noting that on the side of R&D another important market failure operates; the so-called *undervaluation effect* which leads to underinvestment.<sup>16</sup> However, the public firm may partially correct this distortion, as it takes into account consumer surplus in its objective function. The main point here is that the distortions on the side of R&D add to the ones on the side of output, thus reinforcing underproduction as well as the inefficiency in the distribution of equilibrium costs. Hence, when the public firm is the follower in R&D, it can tackle underproduction by increasing the level of total output ( $Q^f > Q^l$ ); in addition to this, it can tackle the inefficiency in the distribution of production costs by raising the private firm's production share ( $x_1^f > x_1^l$ ;  $q_1^f > q_1^l$ ). The reverse holds when the public firm is the R&D leader and therefore, it is always true that  $SW^f > SW^l$ .

## 5 Endogenous timing in the R&D decisions

We proceed to identify the SPNE outcomes of our R&D game in the case where the R&D decision process can last up to two time periods ( $T = 1, 2$ ).<sup>17</sup> As is commonplace in the game-theoretic literature, we propose a *candidate* equilibrium configuration  $(t_i, t_j)$ ,  $i \neq j$ ,  $i, j \in \{2, 3\}$  and check whether there is a *unilateral* incentive for a firm to deviate. If no

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<sup>16</sup>See, for instance, Katsoulacos and Ulph (1998).

<sup>17</sup>If firms make their R&D decisions in exactly two time periods, then a sequential-moves R&D game will emerge, with either the public or the private firm in the leader position. Obviously then, the entire game will last for four time periods.

firm can increase her payoff through a deviation, then the proposed configuration can be sustained as a SPNE.

The main results are presented in the next two Propositions:

**Proposition 5** *The configuration where all firms choose simultaneously in the same time period their R&D levels cannot be sustained as a SPNE.*

The next Proposition contains two additional key results:

**Proposition 6** *(i) There is a SPNE outcome where the private firm chooses its R&D level in period 2 and the public firm chooses its own R&D level in period 3; i.e., the following timing holds,  $t_0 = 3$  and  $t_1 = 2$ . (ii) There is a second SPNE where the reverse timing holds; that is,  $t_0 = 2$  and  $t_1 = 3$ .*

Although the current model has a different focus from previous studies, we note that Nishimori and Ogawa (2005) found for an output setting duopoly that the public firm can either play the role of the leader or follower, i.e., there exist two SPNE. Moreover, on the grounds of Propositions 4 and 6, it follows that in equilibrium the public firm may play a desirable R&D role. In relation to this, Pal (1998) and Matsumura (2003) showed that the equilibrium and the desirable role of the public firm for an output setting mixed market may coincide – independently of whether the private firms are foreign or domestic competitors.<sup>18</sup>

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<sup>18</sup>We find that a game in which the R&D decision process lasts for more than two time periods is not very realistic, although previous authors have undertaken such analyses for output setting oligopolies (e.g. Pal, 1998; Jacques, 2003; Lu, 2007). Our main concern is the rather restrictive (implicit) assumptions made therein, that there is no discounting and that firms can produce only once. Admittedly though, addressing these issues within a single model would make the analysis quite complex.

## 6 Concluding remarks

In this paper, we investigated the sequence of the R&D moves in a mixed market. In particular, utilizing two Stackelberg and one Cournot models of fixed timing, we uncovered the desirable R&D role of the firms. We also employed the observable delay game of Hamilton and Slutsky (1990) to study whether the welfare-maximizing and the equilibrium roles coincide.

The foregoing analysis reveals that the desirable distribution of R&D roles occurs when the private firm is the leader, whereas the public firm retains the follower position. Furthermore, when the order of the R&D moves is endogenised, the public firm may play this desirable role. Overall, these results complement previous studies that considered endogenous timing in output – finding that, under certain circumstances, the desirable and equilibrium roles of the public firm may coincide, whether the private competitor is a domestic or foreign firm (see Matsumura, 2003, p. 283).

The fact that the investigation of R&D competition in mixed markets brings about new findings, calls for more caution in public policy design and implementation at an industry or national level. Again, using Norway’s oil industry as an example, Statoil and Norske Shell (with Exxon to a lesser extent) carry out R&D, which is aimed at the development of new energy technologies. In this respect, our study suggests (in a broader perspective) that mixed markets may require some sort of coordination of their R&D activities. While we do not wish to over-emphasize this point, we believe that the design of innovation policies with clear-cut objectives and distribution of R&D roles across the firms may be useful.



**Appendix A** The SPNE solutions of the Cournot R&D game are as follows:

$$\begin{aligned}
q_0^c &= \frac{(-1 + 10k + 28k^2 + 16k^3)(A - c)}{-2 + 3k + 54k^2 + 80k^3 + 32k^4} \\
q_1^c &= \frac{(-1 + 4k)(1 + 6k + 4k^2)(A - c)}{-2 + 3k + 54k^2 + 80k^3 + 32k^4} \\
\pi_0^c &= \frac{kE(A - c)^2}{(-2 + 3k + 54k^2 + 80k^3 + 32k^4)^2} \\
\pi_1^c &= \frac{(1 - 4k)^2 k \Theta (A - c)^2}{(-2 + 3k + 54k^2 + 80k^3 + 32k^4)^2} \\
CS^c &= \frac{2(-1 + 4k + 24k^2 + 16k^3)^2 (A - c)^2}{(-2 + 3k + 54k^2 + 80k^3 + 32k^4)^2} \\
SW^c &= \frac{2\Xi(A - c)^2}{(-2 + 3k + 54k^2 + 80k^3 + 32k^4)^2}.
\end{aligned} \tag{11}$$

where  $E = 1 - 29k - 40k^2 + 284k^3 + 880k^4 + 832k^5 + 256k^6$ ;  $\Theta = 7 + 43k + 80k^2 + 60k^3 + 16k^4$  and  $\Xi = 1 - 4k - 53k^2 + 64k^3 + 900k^4 + 1616k^5 + 1088k^6 + 256k^7$ .

**Appendix B** The SPNE outcomes of the Stackelberg R&D game with a public leader are:<sup>19</sup>

$$\begin{aligned}
x_1^l &= \frac{(-1 + 4k)(1 + 3k + 2k^2)\Phi(A - c)}{\Omega} \\
q_0^l &= \frac{\Delta(A - c)}{\Omega} \\
q_1^l &= \frac{(1 + 6k + 4k^2)N(A - c)}{\Omega} \\
\pi_1^l &= \frac{k(1 - 4k)^2(1 + k)R^3(A - c)^2}{\Omega} \\
CS^l &= \frac{2\Lambda^2(A - c)^2}{\Omega^2} \\
SW^l &= \frac{4M(A - c)^2}{\Omega},
\end{aligned} \tag{12}$$

where  $\Omega = -16 - 57k + 394k^2 + 2604k^3 + 5528k^4 + 5536k^5 + 2688k^6 + 512k^7$ ;

$\Phi = 7 + 36k + 44k^2 + 16k^3$ ;  $\Delta = -9 + 28k + 508k^2 + 1544k^3 + 1968k^4 + 1152k^5 + 256k^6$ ;

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<sup>19</sup>For the sake of brevity, the expression for the profits of the public firm is omitted.

$$N = -7 - 8k + 100k^2 + 160k^3 + 64k^4; R = 7 + 36k + 44k^2 + 16k^3; \Lambda = -8 - 11k + 266k^2 + 1136k^3 + 1696k^4 + 1088k^5 + 256k^6 \text{ and } M = -2 - 5k + 56k^2 + 314k^3 + 468k^4 + 288k^5 + 64k^6.$$

**Appendix C** The SPNE outcomes of the Stackelberg R&D game with a private leader are:

$$\begin{aligned} x_0^f &= \frac{G(A-c)}{k\Gamma} \\ q_0^f &= \frac{T(A-c)}{k\Gamma} \\ q_1^f &= \frac{(1+6k+4k^2)F(A-c)}{k\Gamma} \\ \pi_1^f &= \frac{(1+k)(1+2k-20k^2-16k^3)^2(A-c)}{k\Gamma} \\ CS^f &= \frac{2Z^2(A-c)^2}{k^2\Gamma^2} \\ SW^f &= \frac{2Y(A-c)^2}{k^2\Gamma^2} \end{aligned} \tag{13}$$

where  $\Gamma = -41 - 212k + 436k^2 + 4624k^3 + 10608k^4 + 10944k^5 + 5376k^6 + 1024k^7$ ;  
 $G = -2 - 13k + 4k^2 + 332k^3 + 1296k^4 + 1872k^5 + 1152k^6 + 256k^7$ ;  $T = -3 - 40k - 20k^2 + 848k^3 + 2912k^4 + 3872k^5 + 2304k^6 + 512k^7$ ;  $F = 1 - 14k - 28k^2 + 184k^3 + 320k^4 + 128k^5$ ;  
 $Z = -1 - 24k - 64k^2 + 404k^3 + 2112k^4 + 3328k^5 + 2176k^6 + 512k^7$  and  $Y = -1 + 6k + 539k^2 + 4184k^3 - 4536k^4 - 161952k^5 - 567936k^6 + 337440k^7 + 7366400k^8 + 23116800k^9 + 38684928k^{10} + 40008704k^{11} + 26411008k^{12} + 10878976k^{13} + 2555904k^{14} + 262144k^{15}$ .

**Appendix D** For the sake of brevity, we have set  $k = 1$  in the proofs of Propositions 1 – 4 and Corollaries 1 – 2. It can be readily verified that the same results hold for every value of  $k$ , with  $k > 0$ .

*Proof of Proposition 1:* (i)  $x_0^c - x_0^f = \frac{1176(a-c)}{5470753}$ ;  $x_0^f - x_0^l = \frac{606324(a-c)}{563094451}$  (which imply,  $x_0^c > x_0^f > x_0^l$ ). (ii)  $x_1^c - x_1^l = -\frac{216(a-c)}{2870563}$ ;  $x_1^l - x_1^f = -\frac{1660884(a-c)}{563094451}$ ; (iii)  $q_0^c - q_0^f = \frac{1932(a-c)}{5470753}$ ;  $q_0^f - q_0^l = \frac{69492(a-c)}{563094451}$ ; (iv)  $q_1^c - q_1^l = -\frac{396(a-c)}{2870563}$ ;  $q_1^l - q_1^f = -\frac{397848(a-c)}{563094451}$ . Q.E.D.

*Proof of Corollary 1:*  $X^l - X^c = -\frac{3492(a-c)}{2870563}$ ;  $X^c - X^f = -\frac{15372(a-c)}{5470753}$ . Similarly,  $Q^l - Q^c = -\frac{972(a-c)}{2870563}$  and  $Q^c - Q^f = -\frac{2688(a-c)}{5470753}$ . Q.E.D.

*Proof of Corollary 2:*  $CS^f - CS^c = \frac{7576445184(a-c)^2}{29929138387009}$ ;  $CS^c - CS^l = \frac{1436390496(a-c)^2}{8240131936969}$ . Q.E.D

*Proof of Proposition 2:*  $\pi_1^l - \pi_1^f = \frac{823959936(a-c)^2}{9679030518239}$ ;  $\pi_1^f - \pi_1^c = \frac{7056(a-c)^2}{913615751}$ . Q.E.D.

*Proof of Proposition 3:*  $PS^c - PS^f = -\frac{1447528968(a-c)^2}{8240131936969}$ ;  $PS^f - PS^l = -\frac{103893659771136(a-c)^2}{317075360746991401}$ .

Q.E.D.

*Proof of Proposition 4:*  $SW^c - SW^l = -\frac{648(a-c)^2}{479384021}$ ;  $SW^l - SW^f = -\frac{1840955616(a-c)^2}{18446411120309}$ .

Q.E.D.

*Proof of Proposition 5:* Suppose instead that both firms acting in the same time period can be sustained as a SPNE, i.e.,  $t_0 = 2$  and  $t_1 = 2$ . From Proposition 4 (Prop. 2) we know that the public (private) firm can increase welfare (profit) by acting as a Stackelberg R&D follower. This is a contradiction and hence, a Cournot game in R&D cannot be sustained as a SPNE. The argument is similar if  $t_0 = t_1 = 3$ , since under Cournot competition in R&D each firm receives the lowest possible payoff. Q.E.D.

*Proof of Proposition 6:* In order to prove the claim in Proposition 6 (i) and given Proposition 5, we have to check whether there is a *unilateral* incentive for the private firm to choose its own R&D level in the subsequent time period 3. If the private firm

would do so, then a Cournot game in R&D would arise. From Proposition 2, however, we know that the private firm cannot increase its profit by acting as a Cournot player. Thus,  $t_0 = 3$  and  $t_1 = 2$  can be sustained as a SPNE outcome.

We proceed to prove part (ii) of the Proposition. We have to show that the public firm has no incentive to deviate and choose its R&D level in period 3. If the public firm does so, a simultaneous moves (Cournot) game arises; that, from Proposition 4 decreases welfare. Hence,  $t_0 = 2$  and  $t_1 = 3$  can be sustained as a second SPNE in the observable delay game. Q.E.D.

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